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Singularity formation and bubbling in nonlinear diffusions

A fundamental question in nonlinear evolution equations is the analysis of solutions which develop singularities (blow-up) in finite time or as time goes to infinity. We review recent results on the construction of solutions to certain notable nonlinear parabolic PDE which exhibit this kind of behavior in the form of "bubbling". This means solutions that at main order look like asymptotically singular time-dependent scalings of a fixed finite energy entire steady state. We carry out this analysis for the classical two-dimensional harmonic map flow $u_t = \Delta u + |\nabla u|^2 u$ into the sphere S^2 and the energy-critical semilinear heat equation $u_t = \Delta u + u^{\frac{n+2}{n-2}}$ in $\mathbf{R}^n$, $n \geq 3$.