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Gravitational allocation on the sphere and overhanging blocks

Given n points on the surface of a sphere, how can we partition the sphere fairly among them in an equivariant way? (See Figure 1.) "Fairly" means that each region has the same area. "Equivariant" means that if we rotate the sphere, the solution rotates along with the points. It turns out that if the given points apply a two-dimensional gravity force to the rest of the sphere, then the basins of attraction for the resulting gradient flow yield such a partition—with exactly equal areas, no matter how the points are distributed. Moreover, this partition minimizes, up to a bounded factor, the average distance between points in the same cell. This is joint work with Nina Holden and Alex Zhai. A second topic where potential theory surprisingly appears starts from the classical overhang problem: Given n blocks supported on a table, how far can they be arranged to extend beyond the edge of the table without falling off? With Paterson, Thorup, Winkler and Zwick we showed that an overhang of order $n^{1/3}$ is the best possible; a crucial element in the proof involves an optimal control problem for diffusion on a line segment. In recent work with Laura Florescu and Miklos Racz, we extended the solution of this control problem to higher dimensions, using Green functions and properties of the divisible sandpile established in earlier work with Lionel Levine.