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Counting thick embeddings

Given compact manifolds M and N, how can we estimate the number of isotopy classes of embeddings $M \to N$ satisfying a constraint on geometric complexity? Of course, there is a profusion of possible answers depending on the category, the dimensions of the manifolds, and the chosen measure of complexity. We show that in codimension at least 3 and for simply connected N, the number of smooth embeddings is at most polynomial with respect to a certain C^2 bound. Unlike in the case of high codimension (the so-called metastable range) the bilipschitz constant is not sufficient to obtain any finite bound; this was remarked already by Gromov in 1978. However, it remains unclear whether our measure of complexity is the "best possible"—a notion I will attempt to define.

In the case $N = \mathbb{R}^n$, we can reframe the question in terms of thick embeddings, analogous to the study of thick knots in $\mathbb{R}^3$. Several non-equivalent definitions of thick PL embeddings were given in papers of Gromov–Guth and Freedman–Krushkal; I will discuss possible definitions in the smooth category.

This is joint work with Shmuel Weinberger.