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Doubly-periodic Costas arrays

A *Costas array* of order n is a $n \times n$ permutation array (of 0s and 1s) where the vectors connecting any two 1s are distinct. Costas arrays have optimal *autocorrelation*; hence they have applications in, e.g., RADAR and SONAR systems and for digital communications.

Costas arrays can equivalently be viewed as permutations f on $[n] = \{0, 1, \dots, n-1\}$ such that for $d \in (0, n-1]$, $f(x+d) - f(x)$ are distinct for all $x \in [0, n-d-1]$. If the domain (respectively, codomain) of f is instead considered to be $\mathbb{Z}/n\mathbb{Z}$, the Costas array is *domain-periodic* (respectively, *range-periodic*) modulo n; geometrically, consider the array wrapped around a vertical (respectively, horizontal) cylinder. It is known that no Costas array may be simultaneously domain- and range-periodic modulo n, though the classical Welch construction of Costas arrays provides a map that is domain-periodic modulo $p-1$ and range-periodic modulo p.

We prove a 1993 conjecture of Golomb and Moreno that any array exhibiting the Welch style of dual-periodicity must be indeed be Welch. Doing so, we show the equivalent result that any polynomial $f \in \mathbb{F}_p[x]$ that satisfies: (1) $f(0) = 0$ and (2) $f(xd) - f(x)$ is a permutation for all $d \neq 1$, is a monomial permutation polynomial.

We will also present some generalizations and related open-problems.