Nonlinear and Stochastic Partial Differential Equations
Équations aux dérivées partielles non linéaires et stochastiques
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RENATO CALLEJA, IIMAS-UNAM
Construction of Quasi-Periodic Response Solutions for Forced Systems with Strong Damping

I will present a method for constructing quasi-periodic response solutions (i.e. quasi-periodic solutions with the same frequency as the forcing) for over-damped systems. Our method applies to non-linear wave equations subject to very strong damping and quasi-periodic external forcing and to the varactor equation in electronic engineering. The strong damping leads to very few small divisors which allows to prove the existence by using a contraction mapping argument requiring very weak non-resonance conditions on the frequency. This is joint work with A.Celletti, L. Corsi, and R. de la Llave.

MICHELE COTI-ZELATI, University of Maryland
Stochastic perturbations of passive scalars and small noise inviscid limits

We consider a class of invariant measures for a passive scalar driven by an incompressible velocity field on a periodic domain. The measures are obtained as limits of stochastic viscous perturbations. We prove that the span of the H^1 eigenfunctions of the transport operator contains the support of these measures, and apply the result to a number of examples in which explicit computations are possible (relaxation enhancing, shear, cellular flows). In the case of shear flows, anomalous scalings can be handled in view of a precise quantification of the enhanced dissipation effects due to the flow.

MAGDALENA CZUBAK, University of Colorado at Boulder
The fluids equations on a hyperbolic space.

In this talk we survey results on how the curvature of the underlying domain can affect the solutions of the equations of fluid mechanics. We compare and contrast with the classical counterparts.

SUSAN FRIEDLANDER, University of Southern California
Asymptotics for magnetostrophic turbulence in the Earth's fluid core

We consider the three dimensional magnetohydrodynamics (MHD) equations in the presence of stochastic forcing as a model for magnetostrophic turbulence. For scales relevant to the Earth's fluid core this MHD system is very rich in small parameters. We discuss results concerning the asymptotics of the stochastically forced PDEs in the limit of vanishing parameters. In particular we establish that the system sustains ergodic statistically steady states thus providing a rigorous foundation for magnetostrophic turbulence.

This is joint work with Juraj Foldes, Nathan Glatt-Holtz and Geordie Richards.

DAVID HERZOG, Iowa State University
Scaling and saturation in infinite-dimensional control problems with applications to SPDEs

We discuss scaling methods which can be used to solve low mode control problems for nonlinear partial differential equations. These methods lead naturally to a infinite-dimensional generalization of the notion of saturation, originally due to Jurdjevic and Kupka in the finite-dimensional setting of ODEs. The methods will be highlighted by applying them to specific equations, including reaction-diffusion equations, the 2d/3d Euler/Navier-Stokes equations and the 2d Boussinesq equations. Applications to support properties of the laws solving randomly-forced versions of each of these equations will be noted.

HELENA NUSSENZVEIG LOPES, Universidade Federal do Rio de Janeiro
Critical Regularity for Energy Conservation in 2D Inviscid Fluid Dynamics

We consider the issue of energy conservation for weak solutions of the $2D$ Euler system with an L^p-control on vorticity, for some $p \geq 1$. This is related to the Onsager conjecture, recently established, which states that, for $3D$ flows, energy is conserved if and only if the velocity is $1/3$-Holder continuous. The Onsager critical regularity is valid in any dimension, however, the forward enstrophy cascade expected in turbulent solutions, from Kraichnan's $2D$ turbulence theory, suggests there may be a regularizing effect not seen in $3D$. It is, hence, plausible that there be a (dynamical) mechanism preventing anomalous energy dissipation in $2D$, even for solutions that are not a priori $1/3$ regular, which cannot be seen by simply estimating the energy flux.

We use a direct argument, based on a mollification in physical space, to show that energy of a weak solution is conserved if $\omega = \nabla^\perp \cdot u \in L^{3/2}$. We construct an example of a $2D$ field $u \in B^{1/3}_{3,\infty}$ (an Onsager-critical space), whose $2D$-curl belongs to $L^{3/2-\varepsilon}$, for any $\varepsilon > 0$, such that the energy flux is non-vanishing, thereby establishing sharpness of the kinematic argument. Finally, we prove that any solution to the Euler equations produced via a vanishing viscosity limit from the Navier-Stokes equations, with $\omega \in L^p$, for $p > 1$, conserves energy. We call such solutions *physically realizable*, and we conclude that there is, indeed, a mechanism preventing anomalous dissipation in 2D in Onsager supercritical spaces.

VINCENT MARTINEZ, Tulane University
Applications of asymptotic coupling in hydrodynamic and related equations

In their 1967 seminal paper, Foias and Prodi captured precisely a notion of finitely many degrees of freedom in the context of the two-dimensional (2D) incompressible Navier-Stokes equations (NSE). In particular, they proved that if a sufficiently large low-pass filter of the difference of two solutions converge to 0 asymptotically in time, then the corresponding high-pass filter of their difference must also converge to 0 in the infinite-time limit. In other words, small scales are "eventually enslaved" by the large scales. One could thus define the number of degrees of freedom to be the smallest number of modes needed to guarantee this convergence for a given flow, insofar as it is represented as a solution to the NSE. This property has since led to several developments in the long-time behavior of solutions to the NSE, in addition to finding applications in data assimilation. In this talk, we will discuss various applications of this phenomenon of "asymptotic coupling" in the context of other hydrodynamic and related equations.

KONSTANTIN MATETSKI, University of Toronto
Convergence of general weakly asymmetric exclusion processes

We consider spatially periodic growth models built from weakly asymmetric exclusion processes with finite jump ranges and general jump rates. We prove that at a large scale and after renormalization these processes converge to the Hopf-Cole solution of the KPZ equation driven by Gaussian space-time white noise. In contrast to the celebrated result by L. Bertini and G. Giacomin (in the case of the nearest neighbour interaction) and its extension by A. Dembo and L.-C. Tsai (for jumps of sizes at most three) we don't use the Hopf-Cole transform and work with the KPZ equation using regularity structures. This is a joint work with J. Quastel from the University of Toronto.

DANA MENDELSON, University of Chicago
Probabilistic scattering for the 4D energy-critical defocusing nonlinear wave equation

We consider the Cauchy problem for the energy-critical defocusing nonlinear wave equation on $\mathbb{R}^4$. It is known that for initial data at energy regularity, the solutions exist globally in time and scatter to free waves. However, the problem is ill-posed for initial data at super-critical regularities. In recent years, probabilistic methods have been used to investigate the behavior of solutions in regimes where deterministic techniques fail. We will present an almost sure global existence and scattering result for randomized radially symmetric initial data of super-critical regularity. The main novelties of our proof are the introduction of an approximate Morawetz estimate to the random data setting and new large deviation estimates for the free wave evolution of randomized radially symmetric data.

This talk is based on joint work with Benjamin Dodson and Jonas Luhrmann.

BENOIT PAUSADER, Brown university
Global existence for a wave-Klein-Gordon system

We prove global existence as well as modified scattering for a quasilinear system of wave and Klein-Gordon equation that is a toy model for the stability of Minkowski space for the Einstein equation with massive scalar field. This is Joint work with A. Ionescu.

CAMELIA POP, University of Minnesota
Boundary estimates for a degenerate parabolic equation with partial Dirichlet boundary conditions

We study the boundary regularity properties and derive pointwise a priori supremum estimates of weak solutions and their derivatives in terms of suitable weighted L^2-norms for a class of degenerate parabolic equations that satisfy homogeneous Dirichlet boundary conditions on certain portions of the boundary. In addition we prove suitable boundary Harnack principles for nonnegative solutions. Such equations arise in population genetics in the study of models for the evolution of gene frequencies. Among the applications of our results is the description of the structure of the transition probabilities and of the hitting distributions of the underlying gene frequencies process.

CATHERINE SULEM, University of Toronto
Surface water waves over bathymetry

We examine the effect of a periodic bottom on the free surface of a fluid linearized near the stationary state, and we develop a Bloch theory for the linearized water wave system. This analysis takes the form of a spectral problem for the Dirichlet - Neumann operator of the fluid domain with periodic bathymetry. We find that, generically, the presence of the bottom results in the splitting of double eigenvalues creating a spectral gap. (joint work with W. Craig, M. Gazeau, C. Lacave).

VLAD VICOL, Princeton University
Nonuniqueness of weak solutions to the SQG equation

We prove that weak solutions of the inviscid SQG equations are not unique, thereby answering an open problem posed by De Lellis and Szekelyhidi Jr. Moreover, we show that weak solutions of the dissipative SQG equation are not unique, even if the fractional dissipation is stronger than the square root of the Laplacian. This talk is based on a joint work with T. Buckmaster and S. Shkoller.

KAREN ZAYA, University of Michigan
On Regularity Properties for Fluid Equations

We discuss a newly developed regularity criterion for the three-dimensional Boussinesq equations, which only imposes a condition on the low modes of the velocity u. The key tool in the development of this weaker regularity criterion is linked to the dissipation wave number.