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A Dynamical Shafarevich Conjecture with Portraits

Fix a number field K, a finite set of places S, and integers $N \geq 1$, $d \geq 2$, and $n \geq 0$. We consider pairs (f, X), where $f : \mathbb{P}^N \to \mathbb{P}^N$ is a morphism of degree d defined over K, and $X \subset \mathbb{P}^N(\overline{K})$ is a Galois invariant subset with $\#X = n$ that satisfies $f(X) \subseteq X$. We say that the pair (f, X) has ¡EM¿good reduction outside¡/EM¿ S if f has good reduction outside S and the points in X remain distinct modulo $\mathfrak{p}$ for all finite primes $\mathfrak{p} \notin S$. ¡B¿Conjecture¡/B¿: There is a $C(N, d)$ so that for all $n \geq C(N, d)$, there are only finitely many $\mathrm{PGL}_{N+1}(R_S)$-equivalence classes of pairs (f, X) having good reduction outside S. In this talk, I will sketch a proof of the conjecture for $N = 1$, and discuss a refined version of the conjecture in which one requires that the map $f : X \to X$ have a specified (weighted) graph structure