Arithmetic Dynamics
Dynamique arithmétique
(Org: **Patrick Ingram** (York University), **Holly Krieger** (Massachusetts Institute of Technology) and/et **Thomas Tucker** (University of Rochester))

JASON BELL, University of Waterloo
A Dynamical Mordell-Lang conjecture for coherent sheaves

We discuss work of Dan Rogalski and Sue Sierra on homological transversality and show that conjectures from noncommutative projective geometry lead naturally to a general dynamical Mordell-Lang conjecture for a variety with an associated endomorphism that can be phrased in terms of vanishing of Tor groups. We give an overview of our conjecture and its relation to the ordinary dynamical Mordell-Lang conjecture. We prove our conjecture in the case of projective surfaces with an automorphism or for general varieties with an associated automorphism that lies in an algebraic group. This is joint work with Matt Satriano and Sue Sierra.

ROBERT BENEDETTO, Amherst College
Computing arboreal Galois groups of some cubic polynomials

Let K be a number field, let $f \in k(x)$ be a rational function of degree $d \geq 2$, and let $a \in K$. The roots of $f^n(z) - a$ are the n-th preimages of a under f, and they have the natural structure of a d-ary rooted tree T. The action of Galois gives a representation of the absolute Galois group of K in the automorphism group of T. In many cases, it is expected that the image of this arboreal Galois representation has finite index in the full automorphism group, but in some cases, such as when f is postcritically finite (PCF), the image is known to have infinite index. In this talk, we present some new examples where the arboreal Galois group can be computed completely.

ARACELI BONIFANT, University of Rhode Island
Tongues and Tricorns in a Space of Rational Maps

In this talk we will describe certain hyperbolic components associated with cubic maps which commute with the antipodal map. Joint with X. Buff and J. Milnor.

JOHN DOYLE, University of Rochester
Reduction of dynatomic curves

Given a one-parameter family of polynomial maps, one can define a *dynatomic curve* $Y_1(n)$, which parametrizes maps in the family together with a marked point of period n. In the classical setting of modular curves of level n, the primes of bad reduction are primes dividing n; however, in the analogous dynatomic setting, it is more difficult to predict the primes of bad reduction. I will discuss recent work on the dynatomic curves for the quadratic family $z^2 + c$, where we have new results concerning smooth and irreducible reduction.

This is joint work with H. Krieger, A. Obus, R. Pries, S. Rubinstein-Salzedo, and L. West.

KENNETH JACOBS, Northwestern University
Archimedean Perspectives from non-Archimedean Dynamics

In arithmetic dynamics, one often tries to transfer ideas from classical complex dynamics into a non-Archimedean context. This is a fruitful avenue of research, and many classical constructions carry over nicely to the arithmetic setting. In this talk, we will reverse this trend by discussing the construction of several (apparently new) equivariants in complex dynamics which arise from three equivariants that Rumely introduced in his study of potential good reduction in non-Archimedean dynamics.

JAMIE JUUL, Amherst College
Images of Iterated Polynomials over Finite Fields

We discuss how to bound the size of the image of the n-th iterate of a polynomial over a finite field using results about arboreal Galois representations. The main term in this bound involves the fixed point proportion of the Galois group of the field extension of $\mathbb{F}_q(t)$ obtained by adjoining all pre-images of the transcendental t under the n-th iterate of the polynomial. We give explicit bounds on the fixed point proportion of the group in the cases where this Galois group is an iterated wreath product.

JAN KIWI, Pontificia Universidad Católica de Chile
Irreducibility of complex cubic polynomials with a periodic critical point

The space of monic centered complex cubic polynomials with marked critical points is isomorphic to $\mathbb{C}^2$. For each $n \geq 1$, the locus $\mathcal{S}_n$ formed by all polynomials with a specified critical point periodic of exact period n forms an affine algebraic set. We prove that $\mathcal{S}_n$ is irreducible, thus giving an affirmative answer to a question posed by Milnor. This is a join work with Matthieu Arfeux (PUCV, Valparaiso).

SARAH KOCH, University of Michigan
Realizing postcritical combinatorics

We say that $X \subset \mathbb{P}^1$ is a *postcritical configuration* if there is a rigid postcritically finite map f with $P_f = X$. In this talk, we determine exactly which subsets $X \subset \mathbb{P}^1$ are postcritical configurations, and we investigate the extent to which the combinatorics of the map $f : P_f \to P_f$ can be specified for $X = P_f$. This talk is based on joint work with L. DeMarco and C. McMullen.

DAVID KRUMM, Colby College
Galois groups in a family of dynatomic polynomials

For any polynomial f with rational coefficients and any positive integer n, let $\Phi_{n,f}$ denote the n-th dynatomic polynomial of f. We will discuss the problem of determining all the groups that can occur as the Galois group of $\Phi_{n,f}$ for some quadratic polynomial f. In particular we will use an explicit form of Hilbert's Irreducibility Theorem to give a complete answer in the case $n = 4$, and we will prove a finiteness result for larger values of n.

KATHRYN LINDSEY, University of Chicago
Convex shapes and harmonic caps

Any planar "shape" P can be embedded isometrically as part of a convex surface S in $\mathbb{R}^3$ such that the boundary of P is the support of the curvature of S. In particular, if P is a connected filled Julia set of a polynomial, this can be done so that the curvature distribution of the convex surface is proportional to the measure of maximal entropy on the Julia set. What would the associated convex subset of $\mathbb{R}^3$ look like? What can it tell us about the dynamics of the polynomial? This talk is based on joint work with L. DeMarco.

NICOLE LOOPER, Northwestern University
Arboreal Galois representations with large image

I will discuss several recent results concerning the size of the arboreal Galois representations induced by polynomials. This will include a partial solution to Odoni's Conjecture, as well as new results in the setting of unicritical maps. I will also address how these theorems relate to the arithmetic of critical orbits.

MICHELLE MANES, University of Hawaii at Manoa
Dynamical Belyi maps

We study the dynamical properties of a large class of rational maps on $\mathbb{P}^1$ with exactly three ramification points, answering a question of Silverman about the number of conservative maps of degree d defined over $\mathbb{Q}$. We explicitly construct two infinite families of examples of such maps. Rather precise results on the reduction of these maps yield strong information on the $\mathbb{Q}$-dynamics.

MYRTO MAVRAKI, University of British Columbia
Quasi-adelic measures, equidistribution and preperiodic points for families of rational maps

Motivated by a question of Zannier, it was shown by Baker and DeMarco that for any fixed complex numbers a and b and integer $d \geq 2$, there are infinitely many $t \in \mathbb{C}$ such that both a and b are preperiodic under iteration by $f_t(z) = z^d + t$ if and only if $a^d = b^d$. Their result was generalized to other 1-parameter families f_t of rational maps by various authors. A key ingredient in their proofs is an arithmetic equidistribution theorem for small points with respect to an adelic measure, proved independently by Baker–Rumely and Favre–Rivera-Letelier.

In this talk we show that most 1-parameter families of rational maps fail to satisfy the adelic hypothesis in the aforementioned equidistribution theorem. We generalize the notion of an adelic measure to that of a quasi-adelic measure and present an equidistribution theorem for quasi-adelic measures. We then connect our work back to questions arising in the theme of unlikely intersections and to an old question concerning the variation of the canonical height in families of rational maps.

This is joint work with Hexi Ye.

RICARDO MENARES, Pontificia Universidad Católica de Valparaíso
Equidistribution of p-adic Hecke orbits on the modular curve

It is well known that the orbits of Hecke correspondences on the modular curve are equidistributed with respect to the hyperbolic measure. Also, by work of Duke and Clozel-Ullmo, it is known that galois orbits of CM points enjoy the same equidistribution property. Recently, Habegger has used this principle to show that the set of singular moduli that are algebraic units is finite.

In this talk we will present a p-adic analogue of the aforementioned equidistribution property of Hecke correspondences, as well as some partial analogues of the equidistribution of CM points. If time allows it, we will also explain how to inject these results into Habegger's strategy in order to prove that, for certain finite sets S of primes, the set of singular moduli which are S-units is finite.

This is joint work with Sebastián Herrero (Chalmers U. of Technology) and Juan Rivera-Letelier (Rochester U.).

JUAN RIVERA-LETELIER, University of Rochester
A tricothomy for the ergodic theory of ultrametric rational maps

We show that the dynamics of an ultrametric rational map endowed with its equidistribution measure is either expanding, divisorial, or wild. In the expanding case, the equilibrium measure is supported on the set of classical points and it is the unique measure of maximal entropy. In the wild case, the equilibrium measure charges the open set of wild critical points. Finally, the most interesting case is when almost every point in the Julia set is contained on a level set of a divisorial function. The main ingredient in the proof is an ultrametric analog of the Hubbard-Lyubich conjecture on biaccesible points of (complex) polynomial Julia sets, shown independently by Smirnov and Zdunik. This rigidity statement is based on a model dynamics, inspired by the piecewise linear models of interval maps developed by Parry and by Milnor and Thurston. This is a joint work with Charles Favre.

JOSEPH SILVERMAN, Brown University
A Dynamical Shafarevich Conjecture with Portraits

Fix a number field K, a finite set of places S, and integers $N \geq 1$, $d \geq 2$, and $n \geq 0$. We consider pairs (f, X), where $f : \mathbb{P}^N \to \mathbb{P}^N$ is a morphism of degree d defined over K, and $X \subset \mathbb{P}^N(\overline{K})$ is a Galois invariant subset with $\#X = n$ that satisfies $f(X) \subseteq X$. We say that the pair (f, X) has ¡EM¿good reduction outside¡/EM¿ S if f has good reduction outside S and the points in X remain distinct modulo $\mathfrak{p}$ for all finite primes $\mathfrak{p} \notin S$. ¡B¿Conjecture¡/B¿: There is a $C(N, d)$ so that for all $n \geq C(N, d)$, there are only finitely many $\mathsf{PGL}_{N+1}(R_S)$-equivalence classes of pairs (f, X) having good reduction outside S. In this talk, I will sketch a proof of the conjecture for $N = 1$, and discuss a refined version of the conjecture in which one requires that the map $f : X \to X$ have a specified (weighted) graph structure