Singularities and Phase Transitions in Condensed Matter
Singularités de la matière condensée et transitions de phases
(Org: **Lia Bronsard** (McMaster University) and/et **Tiziana Giorgi** (New Mexico State University))

MARIA AGUARELES, Universitat de Girona
Laws of motion for spiral waves in the complex Ginzburg-Landau equation in bounded domains

In this talk we consider multiple spiral wave solutions of the general cubic complex Ginzburg-Landau equation in bounded domains. We shall show our results on the effect of the boundaries on the spirals' motion under homogeneous Neumann boundary conditions for small values of the twist parameter q. Explicit laws of motion for rectangular domains can be derived and we show that the spirals motion becomes exponentially slow for a particular critical relation between the twist parameter and the size of the domain. This is a joint work with Prof. Jon S. Chapman.

STANLEY ALAMA, McMaster University
Energy minimizing patterns for a copolymer model with confinement

We identify the Γ-limit of an energy related to nanoparticle/block copolymer blends, in a limit of a large number of nanoparticles occupying a vanishingly small volume in the copolymer sample. The limiting energy consists of two terms: the perimeter of the interface separating the phases and a confinement term representing the effect of the nanoparticles, which attract one of the two stable phases. Our interest is in studying how the confinement term affects the phase transition morphology. We prove that local minimizers of the limiting energy admit regular phase boundaries and derive the first and second variations of the limiting energy functional. Finally we discuss possible critical and minimizing patterns in two dimensions and how these patterns vary from global minimizers of the purely local isoperimetric problem.

GIACOMO CANEVARI, Mathematical Institute, University of Oxford
Sets of topological singularities for vector-valued maps

In some variational models for condensed matter, the configuration of the material is represented by a vector-valued map u that takes values "close" (in a suitable sense) to a distinguished manifold, representing the energetically preferred local states of the material. This is the case, for instance, in the Ginzburg-Landau model for superconductivity or in the Landau-de Gennes model for nematic liquid crystals. In this context, it is useful to have a tool that captures the relevant topological information associated with u and, at the same time, enjoys compactness properties. In the Ginzburg-Landau theory, this is achieved by the use of the distributional Jacobian of u. We discuss another approach, based on a construction by Hardt, Kinderlehrer and Lin, which applies to the analysis of the Landau-de Gennes model.

TIZIANA GIORGI, New Mexico State University
Switching mechanism in the $B_{1RevTilted}$ phase of bent-core liquid crystals

The $B_{\mathrm{1RevTilted}}$ is a uniformly smectic tilted columnar phase in which the macroscopic polarization can be reorientated via electric fields. To study the effects of the various physical parameters on the reorientation mechanism, we consider a local, and a non-local Landau-de Gennes type energy functional, both used in the physics literature. For the case of large columnar samples, we show that the energies give the same qualitative behavior, with a primary role played by the coefficients of the terms that describe the interaction between polarization and nematic vectors.

DMITRY GOLOVATY, The University of Akron
On minimizers of Landau-de Gennes energy for nematic liquid crystals

I will describe recent results for the variational problem arising in mathematics of nematic liquid crystals. The main goal is to understand the behavior of minimizers of the Landau-de Gennes free energy functional defined over a class of symmetric traceless order parameter tensors. The minimizers are characterized by the presence of topological defects, such as vortices and line defects, as well as interfaces that appear at the onset of the isotropic/nematic transition. The talk will be devoted to the analysis of these singularities for particular nematic configurations.

RADU IGNAT, University of Toulouse, France
A DeGiorgi type conjecture for divergence-free maps

We focus on N-dimensional maps u of vanishing divergence defined on the cylinder $\mathbb{R} \times \mathbb{T}^{N-1}$. They minimize a Modica-Mortola type functional with a nonnegative potential W under a boundary condition at infinity that forces u to make a transition between two zeros of W. We characterize the class of potentials W such that every global minimizer u is one-dimensional. Our method is based on calibrations adapted to divergence-free maps. It is a joint work with Antonin Monteil (Louvain-la-Neuve).

SOOKYUNG JOO, Old Dominion University
Polarization-modulated orthogonal smectic liquid crystals

We study the ferroelectric bistability of polarization-modulated orthogonal smectic liquid crystals which exhibits a bistable response to applied electric field. The opposite anchoring at the stripe boundaries and in-polarization form topological singularities. The free energy of the polarization with electric self-interaction term on one stripe reduces to the Ginzburg-Landau functional with boundary penalty term. We describe the boundary vortices of the reduced functional and obtain convergence results of minimizers. We also perform numerical simulations to illustrate the results of our analysis. This is a joint work with T. Giorgi and C. J. García-Cervera.

XAVIER LAMY, Université Toulouse 3
Lifting line fields to director fields with bounded variation

Defects in liquid crystals correspond to singularities of line fields : maps with values into the projective plane. Lifting a line field to a director field amounts to specifying an orientation : a director field takes values into the sphere. We show that any line field with bounded variation (BV) can be lifted to a BV director field. Moreover we obtain an optimal estimate for the BV seminorm of the lifting. This is joint work with Radu Ignat.

OLEG LAVRENTOVICH, Liquid Crystal Institute, Kent State University
"Singularities and Flows in Living Liquid Crystals"

Coupling of activity and orientational order is a quintessential feature of many dynamic out-of-equilibrium systems. We discuss an experimental system, the so-called living liquid crystal, in which the activity and orientational order can be controlled independently of each other. The system represents a dispersion of swimming bacteria (B. Subtilis) in a lyotropic chromonic liquid crystal [1]. The activity of bacteria causes a cascade of orientational instabilities leading to turbulence with nucleating singularities of the director field. In its turn, nematic ordering imposes limitations on the dynamical behavior of the bacteria and their spatial distribution, by concentrating the bacteria around the cores with positive topological charge and depleting the neighborhoods of negative topological charge. Mixed splay-bend director deformations force unidirectional threshold-less flow of bacteria [2]. The work is supported by NSF DMS-1434185, NSF DMR-1507637 and by the Petroleum Research grant PRF 56046-ND7 administered by the American Chemical Society.

1 Zhou, S., Sokolov, A., Lavrentovich, O. D. and Aranson, I. S. Living liquid crystals. P Natl Acad Sci USA 111, 1265-1270, doi:10.1073/pnas.1321926111 (2014). 2 Peng, C., Turiv, T., Guo, Y., Wei, Q.-H. and Lavrentovich, O. D. Command of active matter by topological defects and patterns. Science 354, 882-885, doi:10.1126/science.aah6936 (2016).

DUVAN A. HENAO MANRIQUE, Pontificia Universidad Catolica de Chile
Existence of minimizers of the neoHookean energy in 3D

We show the existence of weak solutions to the elastic equilibrium equations (in their energy-momentum form) for 3D neo-Hookean materials occupying a hollow reference configuration when subject to axisymmetric Dirichlet data. This is the first general existence result for the borderline quadratic energy in 3D. It is joint work with Rémy Rodiac.

LIDIA MRAD, University of Arizona
Dynamic Analysis of Chevron Structures in Smectics

Smectic liquid crystals trapped between two flat substrates exhibit V-shaped defect structures called chevrons under the effect of an applied electric or magnetic field. We consider two models, smectic-A under the effect of a magnetic field using de Gennes energy model and smectic-C under the effect of an electric field using Chen-Lubensky energy model. To understand chevron formation, we analyze the dynamics of these systems. We construct a discretized-in-time gradient flow through energy minimization and prove existence and uniqueness of the continuous gradient flow.

RÉMY RODIAC, Pontificia Universidad Católica de Chile
Regularity of limiting vorticities of the Ginzburg-Landau equations

In this talk I will present some results about the regularity of limiting vorticities of solutions of the Ginzburg-Landau equations with magnetic field. The support of these limiting vorticities gives the limiting repartition of the vortices of a solution of the GL equations. Critical conditions satisfied by these limiting vorticities have been derived by Sandier-Serfaty. These conditions lead us to study some PDE's with measure data for some functions h which also satisfies an inner variational equations. We will show that under some supplementary hypothesis these conditions implies that the support of a limiting vorticity is locally a curve or a set of full Lebesgue measure.

PETER STERNBERG, Indiana University
Phrase transitions in a model for nematic liquid crystals

I will discuss results on a model for nematic-isotropic phase transitions within the general framework of Landau deGennes theory.

IHSAN TOPALOGLU, Virginia Commonwealth University
Height-constrained nonlocal interactions energies via degenerate diffusion

Height-constrained nonlocal interaction energies have recently appeared in models of collective behaviour such as biological swarming and pedestrian crowd motion. The minimization of these energies can also be seen as a "liquid-solid" phase transition where a mixture of two states is possible. Here the additional hard height constraint on admissible functions poses significant challenges both analytically and numerically, and, in order to overcome these challenges, we consider a regularization of the energies by including a highly degenerate diffusion term and approximate the height-constrained model by the unconstrained ones. Justifying our approach analytically in the context of Γ-convergence we implement this scheme numerically in two dimension, and compute steady states via particle approximations. This is a joint project with Katy Craig.

ARGHIR DANI ZARNESCU, BCAM, Basque Center for Applied Math and IMAR, Institute of Mathematics of Romanian Academy
On the dynamical emergence of nematic defects

We consider the effect that the flow has on the dynamics of liquid crystals molecules, within the Q-tensor formalism, and the Beris-Edwards model. We study the limit of high Ericksen number and also the preservation of eigenvalues. We identify some of the flow mechanisms responsible for the appearance of localized gradients that increase in time. This is joint work with Hao Wu (Fudan) and Xiang Xu (Old Dominion).